Supporting Online Material

Data Analysis Procedure

The time-domain signal measured by the sampling oscilloscope is the sum of a voltage step transmitted through the device and an oscillatory signal due to magnetic dynamics. We subtract the background signal due to the transmitted voltage pulse in order to study the magnetic dynamics. Each of our measurements begins with the two magnetic moments in the low-resistance configuration (with the relative angle between the magnetic moments $\theta_0 < 90°$), and we apply steps of positive polarity to rotate the free-layer moment away from the fixed layer moment, exciting dynamics. We can make an approximate determination of the transmitted voltage step by repeating the same experiment at H=0 with the two moments starting in the high-resistance state ($\theta_0 > 90°$), for which positive currents do not excite steady-state dynamics. We then subtract this baseline signal to isolate the component due to the magnetic dynamics. The background subtraction is not perfect, in that it can leave a remnant artifact with a frequency much smaller than the precessional frequency of the free layer. We can eliminate this remnant, if desired, by determining the upper and lower envelope functions of the oscillatory component, and subtracting the average of the two envelope functions. (This procedure was applied to the data in Figs. 1C, 3B, and 3C of the paper, but not Fig. 2.) Our data analysis therefore involves only background subtraction that does not affect the amplitudes or frequencies of the oscillatory signals.

Calculations of Signal Amplitudes

We can estimate the maximum amplitude of the precession, θ_{max}, and also the angle θ_{mis} between the precession axis of the free-layer moment and the fixed direction of the pinned-layer moment, by analyzing the integrated power in the first and second harmonics of the microwave signal that is emitted by the sample when it is biased by a dc current. This estimate assumes that the free layer moves as a single macrospin. The argument generalizes a result presented in ref. [S1].

We make the rough approximation that the motion of the free layer moment (relative to the fixed layer moment) can be described by

$$\theta(t) = \theta_{\mathrm{mis}} + \theta_{\mathrm{max}} \sin(\omega \cdot t) \qquad \text{(S1)}$$

and that the associated voltage signal due to giant magnetoresistance is

$$\Delta V(t) = \frac{I \cdot \Delta R_0}{2} \left[1 - \cos(\theta(t))\right]. \qquad \text{(S2)}$$

Here ΔR_0 is the difference in resistance between antiparallel and parallel alignment of the two magnetic moments and I is the dc current bias. By expanding these expressions using trigonometric identities and keeping only the contributions to the first and second harmonics, we find

$$\Delta V(t) \sim \frac{I \cdot \Delta R_0}{2} \left[2 J_1(\theta_{\mathrm{max}}) \sin(\theta_{\mathrm{mis}}) \sin(\omega t) - 2 J_2(\theta_{\mathrm{max}}) \cos(\theta_{\mathrm{mis}}) \cos(2\omega t)\right], \qquad \text{(S3)}$$

where $J_n(\theta)$ are Bessel functions of the first kind. With this result, the power in the first and second harmonics delivered to the preamp of the spectrum analyzer can be calculated using the equivalent circuit shown in Fig. S1.

$$P_1 = \frac{1}{2}\left(I \cdot \Delta R_0 \cdot J_1(\theta_{\max})\sin(\theta_{\text{mis}})\right)^2 \frac{50\Omega}{\left(2R_S + R_T + R_B + 50\Omega\right)^2}, \tag{S4}$$

$$P_2 = \frac{1}{2}\left(I \cdot \Delta R_0 \cdot J_2(\theta_{\max})\cos(\theta_{\text{mis}})\right)^2 \frac{50\Omega}{\left(2R_S + R_T + R_B + 50\Omega\right)^2}. \tag{S5}$$

For the bias conditions corresponding to Fig. 1C of the paper (I = 8.4 mA, H = 630 Oe), we measure P_1 = 183 pW and P_2 = 8.2 pW, with ΔR_0 = 0.15 Ω, R_S = 5 Ω, R_T = 10 Ω, and R_B = 7 Ω. By solving Equations S4 and S5 numerically, we estimate $\theta_{\text{mis}} \approx 36°$ and $\theta_{\max} \approx 34°$.

Using the same equivalent circuit shown in Fig. S1, the voltage signal $\Delta V_{\text{out}}(t)$ measured in the time domain at the preamp of a sampling oscilloscope is related to $\Delta V(t)$ generated at the sample according to

$$\Delta V_{\text{out}}(t) = \frac{50\Omega}{2R_S + R_T + R_B + 50\Omega} \cdot \Delta V(t). \tag{S6}$$

From this expression, we find that the amplitude of the time-domain voltage oscillation shown in Fig. 1C of the paper is approximately 80% of the value that would correspond to the power measured in the frequency domain. By comparing to numerical simulations of the magnetic dynamics, we believe that this difference is a consequence of averaging over small thermal fluctuations in the initial magnetic-moment orientation at 40 K. Because of these fluctuations, the initial phase of the magnetic oscillations is not precisely the same for each repetition of the measurement, and therefore the average signal measured by the sampling oscilloscope has a slightly smaller amplitude than would an individual trace.

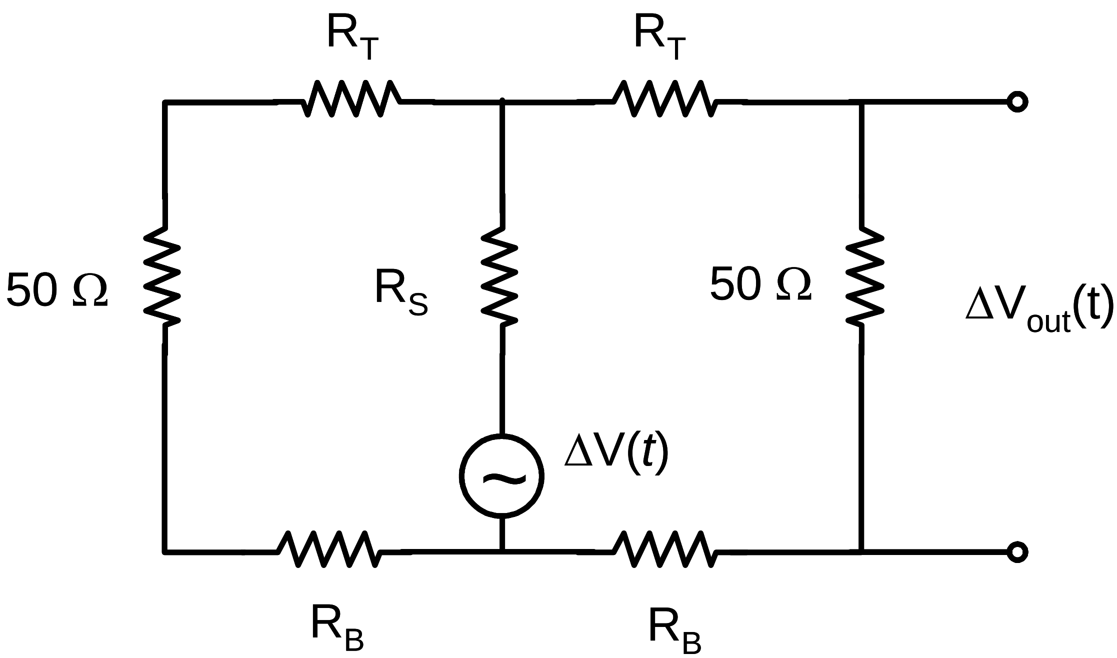

Figure S1. Equivalent circuit for calculating the coupling of time-dependent signals from the sample ΔV(t) to an external measuring device $\Delta V_{out}(t)$. The sample resistance $R_S = 5\ \Omega$, the top-contact resistances $R_T = 10\ \Omega$, the bottom-contact resistances $R_B = 7\ \Omega$, and the 50 Ω resistors represent connections to high-frequency probes.

[S1] S. I. Kiselev *et al.*, *Nature* **425**, 380 (2003).