Ensemble Studies of Nonlinear Conductance Fluctuations in Phase Coherent Samples

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We discuss the voltage dependence of the differential conductance fluctuations in metal samples containing a disordered region much smaller than the inelastic diffusion length at low voltage. Measurements are made for an ensemble of different defect configurations within the same sample. We find that the root mean square amplitude of differential conductance fluctuations within a phase coherent sample is independent of voltage, in conflict with existing theory. The measured voltage correlation scale is at least an order of magnitude smaller than the Thouless energy divided by e.

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Aperiodic fluctuations in the electrical conductance of small disordered metallic samples have been studied thoroughly as a function of magnetic field, Fermi energy, and defect reconfiguration [1,2]. These fluctuations are generally well explained, in terms of quantum interference of the electron wave function, by the theory of universal conductance fluctuations [3]. However, the status of predictions [4,5] pertaining to the voltage (V) dependence of conductance fluctuations (CFs) in mesoscopic samples is not as clear-cut. The interpretation of previous experimental results [6,7] has been complicated because heating effects and dephasing influenced the V dependence of the conductance. In this Letter we discuss disordered metal constrictions connected to clean leads, made so that the region within which interference effects contribute to the conductance is much shorter than the electron dephasing length at low V. We are thus able to make a comparison to theory without the complications of dephasing. We find that existing theory predicts neither the correct V dependence of the amplitude nor the correct V correlation scale of the CFs.

Assuming the absence of inelastic scattering and that many-body effects are unimportant, the current in a sample biased with voltage V may be written as [8]

$$I(V)=\int \frac{dE}{e}\left[G_0+\frac{e^2}{h}F(E,V)\right]\left[f\left(E-E_F-\frac{eV}{2}\right)-f\left(E-E_F+\frac{eV}{2}\right)\right]. \tag{1}$$

Here G_0 is a constant average conductance, $f(E)=[\exp(E/kT)+1]^{-1}$ is the Fermi function, and $F(E,V)$ describes the fluctuations in transmission through the sample for electrons of energy E, when the sample is biased at voltage V. The fluctuating part of the differential conductance is

$$\Delta g_{AC}(V)\equiv dI(V)/dV-G_0=-\frac{e^2}{2h}\int F(E,V)\left[f'\left(E-E_F-\frac{eV}{2}\right)+f'\left(E-E_F+\frac{eV}{2}\right)\right]dE$$
$$+\frac{e^2}{h}\int \frac{dE}{e}\frac{\partial F(E,V)}{\partial V}\left[f\left(E-E_F-\frac{eV}{2}\right)-f\left(E-E_F+\frac{eV}{2}\right)\right]. \tag{2}$$

Larkin and Khmel'nitskii (LK) [5] assert, in effect, that $F(E,V)$ is a random function in both E and V having a root mean square (rms) amplitude of order 1, with an energy correlation scale of order E_{Th} and a V correlation scale which is also assumed to be of order E_{Th}/e, where $E_{Th}=\pi^2\hbar v_F l/3L^2$ is the Thouless energy [3]. Here v_F is the Fermi velocity, l is the mean free path, and L is the length of the phase coherent sample, assumed to be diffusive. We consider temperatures $T<E_{Th}/k_B$. It follows then that the measured V correlation scale of the CFs should be $V_c\sim E_{Th}/e$. For $V\ll V_c$, the rms antisymmetrized conductance, averaged over magnetic field or defect configurations, should grow linearly with V [4],

$$\left\langle\left[\frac{\Delta g_{AC}(V)-\Delta g_{AC}(-V)}{2}\right]^2\right\rangle^{1/2}\sim\frac{e^2}{h}\frac{V}{V_c}. \tag{3}$$

For $V\gg V_c$, Δg_{AC} is calculafed to be dominated by the last integral in Eq. (2). Estimating the integral as a sum of random contributions of order 1 from V/V_c uncorrelated energy intervals, LK [5] predict that the rms fluctuations in differential conductance should grow with V as

$$\langle[\Delta g_{AC}(V)]^2\rangle^{1/2}\sim(e^2/h)(V/V_c)^{1/2}. \tag{4}$$

We note that the calculation of LK assumes a cylindrical sample with $l\ll L$, but they state that modifications of the sample shape or l/L will not change their result [5]. For samples with a three-dimensional shape, Lee, Stone, and Fukuyama have shown that the energy correlation function decays so slowly that energies differing by more than E_{Th} are not effectively independent [3]; in this case, the rms value of Δg_{AC} would grow even more quickly when $V\gg V_c$ than in Eq. (4).

Previous experiments have not yet provided an unambiguous test of these predictions. Direct measurements of

differential CFs in both metal [6] and semiconductor [7] wires have found CFs which decreased in amplitude, rather than increased, with increasing V. However, these experiments also observed correlation scales which increased with V, a sign that the dephasing length was decreasing. These results thus cannot be compared directly to the LK theory, which assumes a phase coherent sample. Measurements of the V correlation scale of CFs in semiconductors have generally been in good agreement with the Thouless scale [9], but in metal wires the measured scale has been much smaller than predicted theoretically [6]. It is not clear whether this might be related to dephasing. CFs in point contacts have been measured with rms amplitude roughly constant at low V, but Holweg *et al.* suggested that this discrepancy with theory was due to the ballistic nature of the samples [10].

We have conducted systematic measurements of CFs in disordered samples which are smaller than the electron phase coherence length over much of the V range studied. We make these samples by first producing ballistic Cu constrictions [11]. We use electron beam lithography to form, in a silicon nitride membrane, a tapered hole whose minimum diameter d may range from 3 to 15 nm. We then rotate the membrane to expose both sides while evaporating Cu in high vacuum to fill the hole and form the constriction. As fabricated, these devices are highly ballistic, with diameters much smaller than the electron mean free path in the bulk Cu film at 4.2 K, 180 nm. As a result, the magnitude of CFs due to electron interference is greatly reduced below the "universal" value $\sim e^2/h$; a typical rms amplitude is $0.02e^2/h$ [10,12].

To make a disordered region, we add defects to an existing clean nanobridge using electromigration. This is done by applying a high V (100–500 mV) across the sample so that Cu atoms are moved within the constriction. Atoms rearranged near the constriction region are not perfectly ordered, so this technique allows us to change the mean free path l near the constriction; however, the electrodes remain clean because electromigration occurs only where the current density and electric field strength are highest. A schematic cross section of the device after electromigration is shown in the inset to Fig. 1(b).

After electromigration has added disorder to the constriction region, the amplitude of the CFs at low V increases to be of order e^2/h, an indication that l is comparable to d. The magnetic field (B) dependence of the CFs is very weak, showing that the size of the region within which the interference effects are generated is small. Figure 1(a) shows the CFs from a 105 Ω disordered Cu constriction in several B fields. The CFs are field sensitive, but the correlation scale, B_c, is greater than the highest available field, 2.6 T. B_c is expected [1] to be approximately $2\phi_0/L^2$, where ϕ_0 is the flux quantum and L^2 is the effective cross-sectional area of the sample. A lower bound of 2.6 T on B_c corresponds to an upper bound of $L\sim 40$ nm for the scale of the region near the constriction within which interference effects are important.

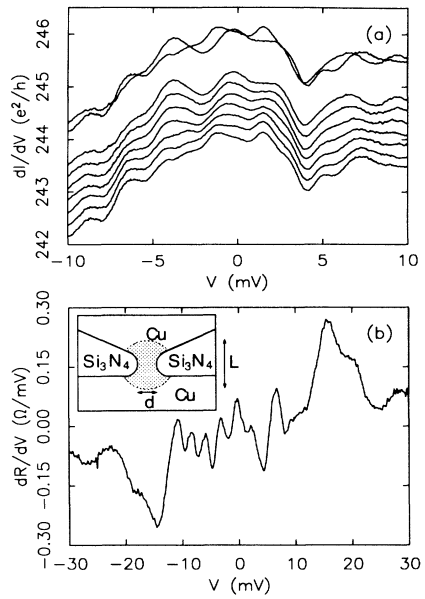

FIG. 1. (a) Lower seven curves: Differential CFs for a disordered Cu sample at 4.2 K, at equal increments of magnetic field from 0 to 2.6 T, bottom to top. Field is parallel to the constriction axis, the orientation of greatest field sensitivity. Curves are offset by $0.2e^2/h$. Upper two curves: CFs at 0 and 2.6 T with 0 relative offset. (b) PCS for the disordered Cu sample at 1.8 K. Inset: Device schematic after electromigration. L is the scale of the disordered Cu region within which interference effects are generated; d is the constriction diameter. The disorder may not be homogeneous or have an abrupt boundary.

Other device parameters may be estimated from the second derivative of the I-V characteristic for the sample, shown in Fig. 1(b). The aperiodic oscillations at low V are CFs, while the antisymmetric structure at 15 mV and beyond is primarily due to scattering by single phonon emission. Such phonon scattering has long been studied by point contact spectroscopy (PCS) [13]. Elastic scattering in the constriction reduces the PCS phonon signal [14]. The size of the first phonon peak in Fig. 1(b) at 15 mV, 0.26 Ω/mV, can be compared to that found for ballistic nanobridges of comparable resistance, 0.38 Ω/mV [12]. This factor of 0.7 reduction indicates a mean free path $l\sim 2d$, with perhaps a factor of 2 uncertainty due to the fact that the constriction may not have the ideal geometry [14]. This estimate, together with the formula for the resistance of a point contact [13],

$$R \approx (h/e^2)[8/(k_F d)^2][1+0.4d/l], \qquad (5)$$

allows estimates for d and l in the constriction region: $d\sim 3.6$ nm and $l\sim 7$ nm.

Our samples therefore do not precisely realize the idealized samples assumed in the explicit calculation of LK. Our samples are not cylinders, and l is comparable to L. However, because LK [5] state that the shape of the contacts is "not important" and the condition $l \ll L$ is

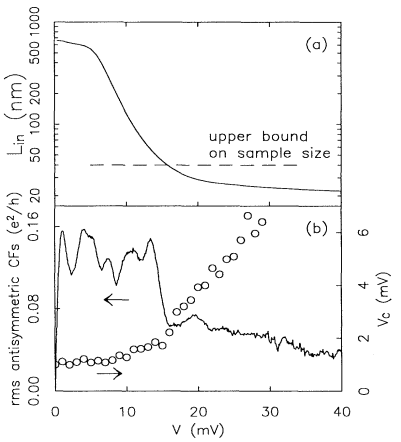

FIG. 2. (a) Electron-phonon inelastic diffusion length for the Cu sample, calculated as described in the text. The line at 40 nm marks the upper bound on the effective sample length given by the magnetic field correlation scale. (b) Line: rms antisymmetric differential CFs for the disordered Cu sample. Circles: Voltage correlation scale of the CFs as a function of V.

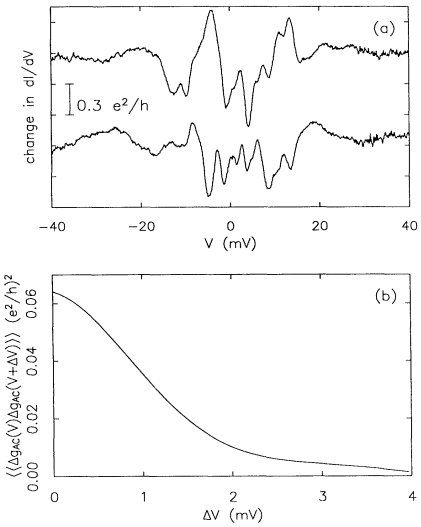

FIG. 3. (a) Two conductance traces from an ensemble of 25 CF patterns for different defect configurations in the disordered Cu device, at 1.8 K. The average pattern is subtracted. (b) Average of the autocorrelation functions for the ensemble, between -10 and 10 mV, which yields $V_c = 1.1$ mV.

"not a restriction," we would expect their results to apply to our CFs as long as phase coherence is maintained.

In the range of T and V of our measurements, the electron dephasing rate is determined by electron-phonon scattering [12,15]. This scattering rate can be measured directly from PCS curves of ballistic Cu point contacts [13]:

$$\frac{1}{t_{in}(eV)} = \frac{3\pi v_F}{4d}\int_0^V \frac{1}{R}\frac{dR(V)}{dV}dV. \quad (6)$$

The inelastic diffusion length [16] is then

$$L_{in} = (2v_F l t_{in})^{0.5}.$$

The value of $L_{in}(eV)$ for the disordered sample, determined using PCS data from a ballistic Cu constriction [17], is shown in Fig. 2(a). At low voltages, L_{in} is hundreds of nm, much longer than the length scale over which quantum interference effects are generated. L_{in} begins to drop sharply near 10 mV, in the range of typical phonon energies, but only for $V > 15$ mV does L_{in} decrease below 40 nm, where dephasing can be expected to affect the conductance of the sample.

With this understanding of the length scales important to our devices, we now analyze the V dependence of the CFs. Our samples have the convenient property that, at low T, by applying a bias of 50–100 mV, lower than that used for electromigration, we can reconfigure defects in the sample without changing the sample geometry significantly [18]. Thus we can perform ensemble averages of quantum interference effects over many realizations of disorder, all within one sample. Figure 3(a) shows two conductance traces from an ensemble of 25 defect configurations obtained in this manner, with the average curve subtracted to remove the phonon signal. The CFs are clearly sensitive to the detailed defect distribution. Below 10 mV, the V correlation scale of the CFs does not increase with V, but beyond 15 mV the amplitude of the CFs drops and the V scale increases. The average of the autocorrelation functions of the conductance curves below 10 mV for the 25 defect configurations, shown in Fig. 3(b), gives $V_c = 1.1$ mV. This is the same order as the thermal smearing of the measurements at $T = 1.8$ K, $3.5kT/e = 0.5$ mV. However, we have cooled samples with similar CFs to 50 mK and have never observed measurable fluctuations with $V_c < 0.5$ mV.

A quantitative measure of the V dependence of V_c is shown in Fig. 2(b). At each V for which a point is shown, each of the 25 conductance curves (with the average subtracted) was antisymmetrized about that V, and the rms average of the resulting curves was computed. The result, for each V, was a curve similar to the continuous curve in Fig. 2(b), the rms V-antisymmetric CFs around $V = 0$. The range over which the antisymmetrized CFs initially rise from 0 to their full value is a measure of V_c at the V in question. This calculation is a generalization of Eq. (3), used to solve for V_c. The proportionality factor required in Eq. (3) was determined by normalizing V_c at low V to the value from the autocorrelation function. Consistent with the raw data, we find that V_c is constant until about 10 mV, and increases sharply beyond 15 mV. This provides additional confirmation that the sample length is less than the dephasing length until about 15 mV.

Unlike previous studies in disordered systems, we are therefore able to study the V dependence of the CF amplitude in a sample for which dephasing is demonstrably not significant until tens of V_c. We now examine in more detail the continuous curve in Fig. 2(b), the rms V-

antisymmetric (around $V=0$) CFs for our ensemble of 25 realizations of disorder. We consider only the antisymmetric part of the CFs to eliminate the possibility of contributions from V-symmetric signals that are not related to quantum interference [19,20]. In contrast to the theory of LK [5], the amplitude of the CFs does not increase for $V > V_c$. The rms amplitude of the CFs is constant for $V_c < V < 15$ mV, at which point dephasing begins to become important. This is the same behavior observed by Holweg *et al.* in ballistic constrictions [10]. Our devices, being much more disordered, show that this constant amplitude is not a property of only ballistic samples, but is a more general property of CFs in phase coherent devices. In terms of theory, it is apparent that the last integral in Eq. (2) does not grow rapidly with V to dominate the behavior of $\Delta g_{AC}(V)$. This could be due to the existence of negative correlations between values of $\partial F/\partial V$ at different E that lead to cancellations in the integral for large V. Such correlations can occur in heuristic pictures in which CFs are due to fluctuations in the number of one-electron eigenstates having energies within ranges of width eV_c [21]. An applied voltage might cause redistributions in the energies of eigenstates, so that a negative value of $\partial F/\partial V$ at one E, due to a loss of eigenstates, would be correlated with positive values due to gains at nearby E.

Our measured value of V_c is also much smaller than expected [5]. Using either the estimate from the insensitivity of the CFs to a B field or the diffusion length at which dephasing becomes significant (Fig. 2), we measure an upper bound on the effective sample length of 40 nm. This implies a lower bound on E_{Th}: $E_{Th} > 15$ meV. The measured value V_c, 1.1 mV, is therefore at least a factor of 10 smaller than E_{Th}/e. This result is similar to measurements in Sb and Au wires [6].

Tang and Fu [22] have argued that the proper V correlation scale is set by the single particle energy level spacing, not the Thouless energy. This proposal agrees with our measured V_c quite well. A Cu sphere of diameter 4.6 nm is required in order to have a single particle energy spacing of 1.1 mV. This size is in good correspondence to our constriction diameter, the length scale over which V drops across the sample. The implications of the model of Tang and Fu for the V dependence of the amplitude of the CFs are uncertain. [Certainly, a careful study of the sample size dependence of V_c in phase coherent samples would be welcome to help determine the physical origin of the observed values of V_c.]

In summary, by making samples consisting of a small disordered region between clean three-dimensional electrodes, we have studied the V dependence of conductance fluctuations in samples much smaller than the electron dephasing length. We find that the amplitude of the differential conductance fluctuations is constant as a function of V as long as phase coherence is preserved, in conflict with existing theory. The V correlation scale of the fluctuations is at least an order of magnitude smaller than the Thouless energy scale.

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